Il nuovo Sistema Internazionale (SI)

Le unità elettromagnetiche

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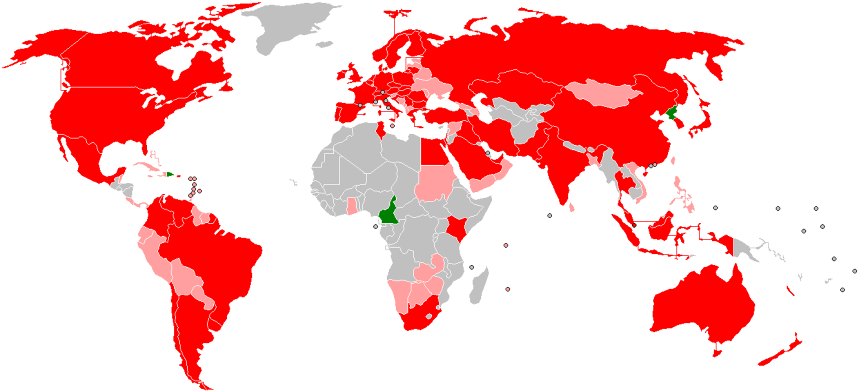
The metre convention, 1875



Signatories of the Metre Convention, 2017

The present SI

Definition of the units

$$X = X\,[X]$$

X	quantity
$\{X\}$	value
$[X]$	unit (reference quantity)

Example: $L = 3\,\mathrm{m}$

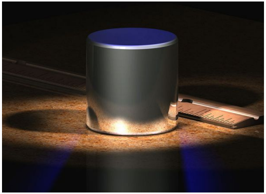
L	quantity
$\{3\}$	value
$[\mathrm{m}]$	unit

The present SI

Units:

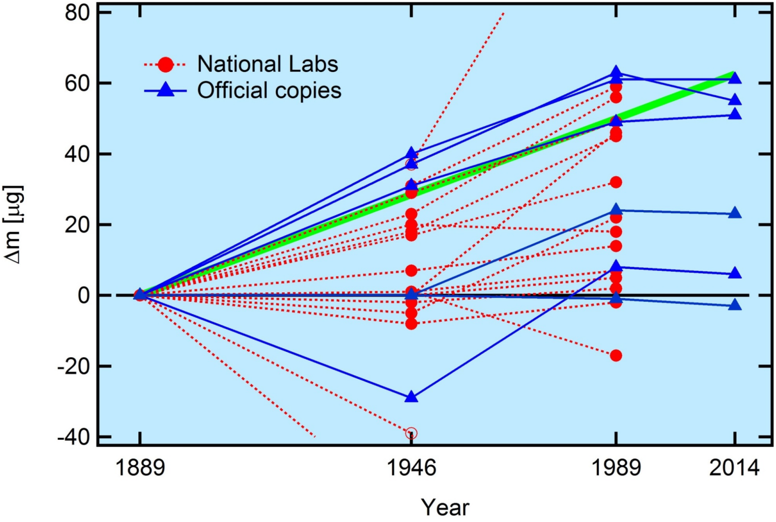
- fundamental constant ($c = 299\,792\,458\,\mathrm{m/s}$)
- property of a material ($T_{\mathrm{TPW}} = 273.16\,\mathrm{K}$);

- property of an artifact ($\mathfrak{K} = 1\,\mathrm{kg}$)



The present SI

The kilogram $\mathfrak{K}$



Suspected drift: 35 µg over 140 y

SI units for electromagnetic quantities

Base units

Symbol	Unit name
m	metre
kg	kilogram
s	second
A	ampere

SI units for electromagnetic quantities

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Derived units

$\mathrm{m}^{\alpha}\,\mathrm{kg}^{\beta}\,\mathrm{s}^{\gamma}\,\mathrm{A}^{\delta}$, where α, β, γ, and δ are usually integers.

SI units for electromagnetic quantities

Derived units with special names

Derived quantity	name	symbol	expression in terms of base units
frequency	hertz	Hz	s^{-1}
energy	joule	J	$m^2\,kg\,s^{-2}$
power	watt	W	$m^2\,kg\;s^{-3}$
electric charge	coulomb	C	$s\;A$
electric potential difference	volt	V	$m^2\;kg\;s^{-3}\;A^{-1}$
electric capacitance	farad	F	$m^{-2}\;kg^{-1}\;s^{-4}\;A^2$
electric resistance	ohm	Ω	$m^2\;kg\;s^{-3}\;A^{-2}$
electric conductance	siemens	S	$m^{-2}\;kg^{-1}\;s^3\;A^2$
magnetic flux	weber	Wb	$m^2\;kg\;s^{-2}\;A^{-1}$
magnetic flux density	tesla	T	$kg\;s^{-2}\;A^{-1}$
inductance	henry	H	$m^2\;kg\;s^{-2}\;A^{-2}$

The ampere

In the present SI, the definition of the base unit ampere is mechanical:

> *The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to* 2×10^{-7} *newton per metre of length.*

From the definitions of the ampere and the metre[1] it follows that these quantities are **exact**:

- $\mu_0 = 4 \times 10^{-7}\,\mathrm{H/m}$ the magnetic constant (permeability of free space);
- $\epsilon_0 = \left(\mu_0 c^2\right)^{-1} = 8.854\,187\,817\ldots\,\mathrm{pF/m}$, the electric constant (permittivity of free space);

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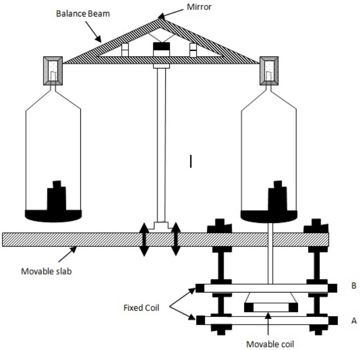
[1] given in terms of the second and of the speed of light in vacuum $c = 299\,792\,458\,\mathrm{m/s}$

Realization of the ampere

The (electrodynamic) ampere balance



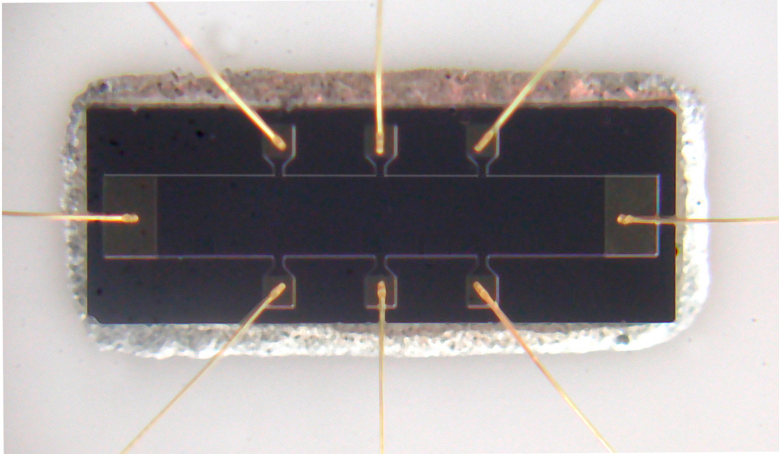
Ampère force law: $F = \mu_0 k I^2$ where k (adimensional) is computed from geometry.

Quantum electrical metrology experiments

Macroscopic quantum effect that display an electrical quantity related to fundamental constants

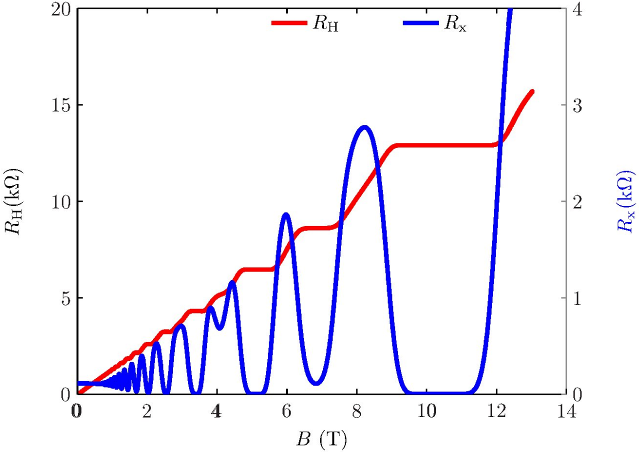
- quantized resistance: the quantum Hall effect
- quantized flux counting: the Josephson effect
- quantized charge counting: single-electron devices

The quantum Hall effect



AlGaAs/GaAs Hall bar heterostructure, 1 mm × 0.4 mm;

The quantum Hall effect



Observation of quantum Hall effect. $T = 2.53\,\mathrm{K}$, $I = 25\,\mu\mathrm{A}$. V_H across two opposite contacts and V_x between two adjacent contacts, is measured as a function of the magnetic field B. The value of the Hall resistance $R_H = V_H/I$ and of the longitudinal resistance $R_x = V_x/I$. Hall plateaus corresponding to $R_H = R_K/i$ (with $i = 2, 4, 6...$) are clearly visible; correspondingly, $R_x \approx 0$.

The quantum Hall effect

According to theory each plateau i is centered on magnetic field values $B_i = n_s h / i\, e$, and has a resistance value R_K/i, where R_K is the *von Klitzing constant*:

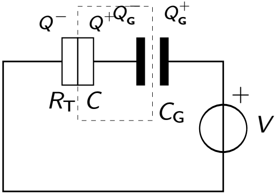
$$R_K = \frac{h}{e^2} = \frac{\mu_0\, c}{2\alpha}. \tag{1}$$

R_K is thus linked to the fine structure constant α, which can be measured by non-electrical means.
The CODATA least-squares adjustment, which embeds Eq. (1) as an exact physical law, gives the value

$$R_K = 25\,812.807\,443\,4(84)\,\Omega \quad [3.2 \times 10^{-10}].$$

Quantized charge counting

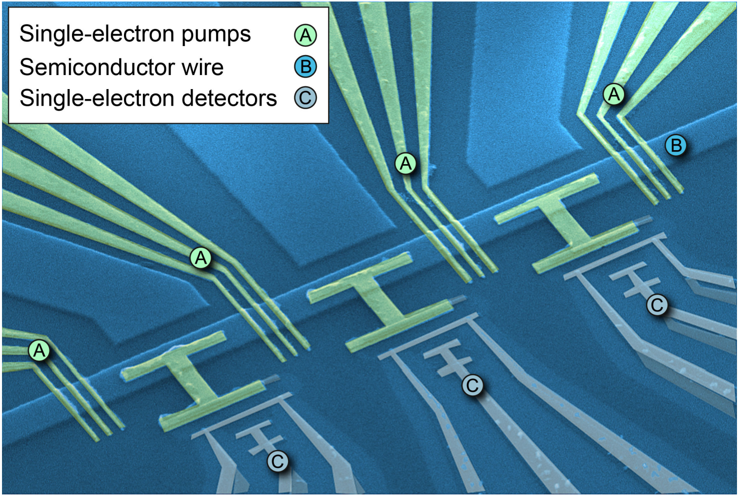
Single charge confinement



Single-electron box, coupled to an external circuit with a tunnel junction (with tunnel resistance R_T and capacitance C) and a capacitor C_G.

Quantized charge counting

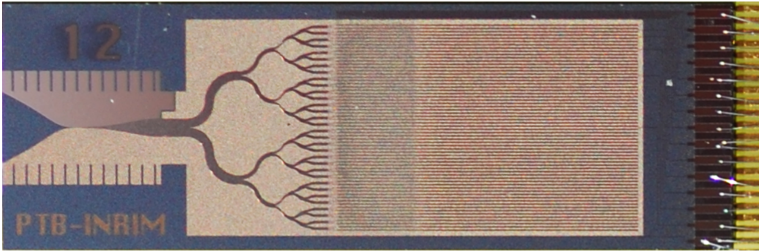
Nanodevices



Semiconductor single-electron pumps (PTB).

Counting flux quanta

Josephson binary DAC



A Josephson junction binary array chip.

- $V_{\mathrm{dc}} = n\Phi_0 f_{\mathrm{ac}} = \dfrac{n f_{\mathrm{ac}}}{K_{\mathrm{J}}}$
- $K_{\mathrm{J}} = 2\dfrac{e}{h} = 483\,597.8525(30)\,\mathrm{GHz/V}$ is the Josephson constant

The quantum experiments in the present SI

In 1989 (Taylor and Witt, 1989)

$$K_{\mathrm{J}} = 483\,597.9(2)\,\mathrm{GHz/V} \qquad [4 \times 10^{-7}]$$

$$R_{\mathrm{K}} = 25\,812.807(5)\,\Omega \qquad [2 \times 10^{-7}]$$

but the *reproducibility* of Josephson and quantum Hall experiments in different experiments and different laboratories had much better uncertainty (10^{-9}–10^{-10}).

Solution: invent non-SI units!

$$K_{\mathrm{J\text{-}90}} = 483\,597.9\,\mathrm{GHz/V} \qquad [\mathrm{exact}]$$

$$R_{\mathrm{K\text{-}90}} = 25\,812.807\,\Omega \qquad [\mathrm{exact}]$$

18th CGPM resolution 6: Valid since January 1, 1990.
To the choice of $K_{\mathrm{J\text{-}90}}$ and $R_{\mathrm{K\text{-}90}}$, units $\mathit{\Omega}_{90}$, H_{90}, F_{90}, A_{90}, W_{90} are associated[2]. These are the electrical units still in use nowadays.

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[2] The conventional units are written in italic type in recognition of the fact that they are physical quantities.

The quantum experiments in the present SI

Present status of the conventional units

Becuase of improvements in the measurement of fundamental constants, today (CODATA 2014)

$$K_\mathrm{J} = 483\,597.8525(30)\,\mathrm{GHz/V} \qquad [6.1 \times 10^{-9}]$$

$$R_\mathrm{K} = 25\,812.807\,455\,5(59)\,\Omega \qquad [2.3 \times 10^{-10}]$$

Hence

$$V_{90} = 1 + 9.8(6) \times 10^{-8}\,\mathrm{V}$$

$$\mathit{\Omega}_{90} = 1 - 1.764(2) \times 10^{-8}\,\Omega$$

The forthcoming SI

Redefinition of the SI base of interest for electromagnetism:

- kg the kilogram;
- A the ampere;

by fixing the values of the fundamental constants:

- h Planck constant;
- e elementary charge;

The forthcoming SI: the base unit ampere

The ampere will be redefined as:

> *The ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge e to be $1.602\,176\,621 \times 10^{-19}$ when expressed in the unit C, which is equal to A s, where the second is defined in terms of $\Delta\nu_{\mathrm{Cs}}$.*

The kilogram will be redefined as:

> *The kilogram, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant h to be $6.626\,070\,040 \times 10^{-34}$ when expressed in the unit J s, which is equal to $\mathrm{kg\,m^2\,s^{-1}}$, where the metre and the second are defined in terms of c and $\Delta\nu_{\mathrm{Cs}}$.*

The forthcoming SI: realization of the units

Consequences of the redefinition

e is **exact**;

$\Rightarrow$ any electron-counting experiment is a realization of the ampere;

$R_\mathrm{K} = \dfrac{h}{e^2}$ is **exact**;

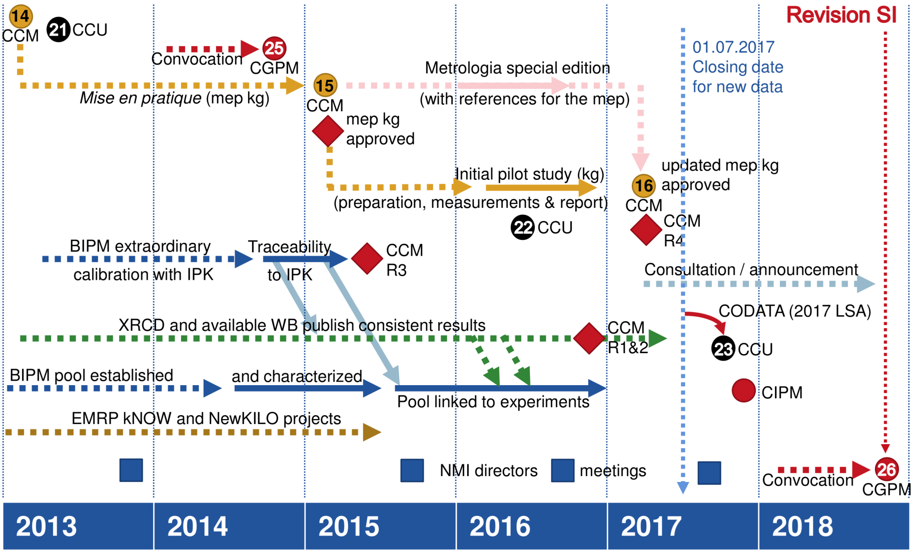
$\Rightarrow$ the quantum Hall effect is a **realization** of the ohm;

$K_\mathrm{J} = \dfrac{2e}{h}$ is **exact**;

$\Rightarrow$ the Josephson effect will be a **realization** of the volt;

$\Rightarrow$ Combining Josephson and quantum Hall effects with Ohm's law is a realization of the ampere.

Joint CCM and CCU roadmap for the new SI



Revision SI

14 CCM

21 CCU

Convocation

25 CGPM

Mise en pratique (mep kg)

15 CCM

Metrologia special edition

(with references for the mep)

01.07.2017
Closing date
for new data

mep kg
approved

Initial pilot study (kg)

(preparation, measurements & report)

updated mep kg
approved

16 CCM

22 CCU

CCM
R4

BIPM extraordinary
calibration with IPK

Traceability
to IPK

CCM
R3

Consultation / announcement

CODATA (2017 LSA)

XRCD and available WB publish consistent results

CCM
R1&2

23 CCU

BIPM pool established

and characterized

Pool linked to experiments

CIPM

EMRP kNOW and NewKILO projects

NMI directors meetings

Convocation

26 CGPM

2013 2014 2015 2016 2017 2018

CCEM Guidelines

CCEM/17-08	*Mise en pratique* for the ampere and other electric units in the International System of Units (SI)
CCEM/17-09	Guidelines for Implementation of the Revised SI

CCEM/17-09

Guidelines for Implementation of the Revised SI

- New SI approval date: November 2018
- New SI implementation: May 20, 2019, International Metrology Day

- $1\,V_{90} \Rightarrow 1\,\mathrm{V} \approx 1 - 1 \times 10^{-7}\,V_{90}$
- $A_{90}, W_{90}, \ldots \Rightarrow \mathrm{A}, \mathrm{W}, \ldots$
- $1\varOmega_{90} \Rightarrow 1\,\Omega \approx 1 + 2 \times 10^{-8}\,\varOmega_{90}$
- $F_{90}, H_{90}, \ldots \Rightarrow \mathrm{F}, \mathrm{H}, \ldots$

CCEM/17-09

Guidelines for Implementation of the Revised SI

What do I need to do in preparation for the Revised SI?

- *Familiarize with the new SI*
- *Review your traceability requirements by identifying standards, artifacts, instrumentation, control and statistical software and specific measurements that may be affected by this change. Particular focus should be made on the highest accuracy (lowest uncertainty) components*
- *Review your quality management documents to identify references to the conventional units and conventional values $K_{\mathrm{J}-90}$, $R_{\mathrm{K}-90}$, for example in calibration procedures and measurement software.*

CCEM/17-09

Guidelines for Implementation of the Revised SI

What do I need to do on or immediately following implementation day?
Relative change $d \approx 1 \times 10^{-7}$ *for V, A, W, ...;* $d \approx 2 \times 10^{-8}$ *for Ω, H, F*

$U_\mathrm{r} > 2.5 \times 10^{-7}$ *: no action is necessary*
$U_\mathrm{r} < 2.5 \times 10^{-7}$ *: numerical correction or recalibration. Note it also applies to the software that controls these measurements.*

In Italy, for calibration centers:
Zener reference: $U_\mathrm{r}(10\,\mathrm{V}) = 5 \times 10^{-7} \Rightarrow$ no action.

Thanks!