La ridefinizione del Sistema Internazionale di unità di misura

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The present International System of units (SI)

The seven base units

- m The **metre** is the length of the path travelled by light in vacuum during a time interval of 1/299792458 of a second.
- kg The **kilogram** is the unit of mass; it is equal to the mass of the international prototype of the kilogram.
- s The **second** is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom.
- A The **ampere** is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 m apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.
- K The **kelvin**, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.
- mol The **mole** is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kg of carbon 12.
- cd The **candela** is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian.

SI units for electromagnetic quantities

Derived units with special names

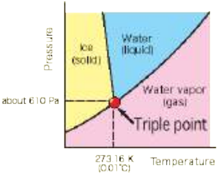
Derived quantity	name	symbol	expression in terms of base units
frequency	hertz	Hz	s^{-1}
energy	joule	J	$m^2\,kg\,s^{-2}$
power	watt	W	$m^2\,kg\,s^{-3}$
electric charge	coulomb	C	$s\,A$
electric potential difference	volt	V	$m^2\,kg\,s^{-3}\,A^{-1}$
electric capacitance	farad	F	$m^{-2}\,kg^{-1}\,s^{-4}\,A^2$
electric resistance	ohm	Ω	$m^2\,kg\,s^{-3}\,A^{-2}$
electric conductance	siemens	S	$m^{-2}\,kg^{-1}\,s^3\,A^2$
magnetic flux	weber	Wb	$m^2\,kg\,s^{-2}\,A^{-1}$
magnetic flux density	tesla	T	$kg\,s^{-2}\,A^{-1}$
inductance	henry	H	$m^2\,kg\,s^{-2}\,A^{-2}$

Definition of units

in the present SI

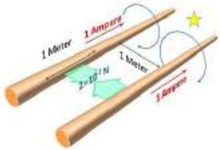
an artefact:

> *The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.*



a natural property

> *The kelvin is the fraction* $1/273.16$ *of the thermodynamic temperature of the triple point of water.*



an idealized experiment

> *The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length [...] would produce a force equal to* 2×10^{-7} *newton per metre of length*

The ampere

In the present SI, the definition of the base unit ampere is mechanical:

> *The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.*

All electromagnetic derived units have an ultimately mechanical definition also.

These quantities are **exact**:

$\mu_0 = 4\pi \times 10^{-7}$ H/m the *magnetic constant*;

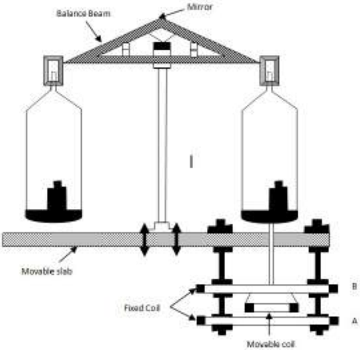
$\epsilon_0 = \left(\mu_0 c^2\right)^{-1} = 8.854\,187\,817\ldots$ pF/m, the *electric constant*

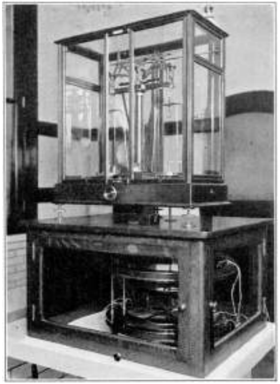
$Z_0 = \mu_0\, c = \sqrt{\mu_0\, \epsilon_0^{-1}} = 376.730\,313\,4\ldots\ \Omega$, the *impedance of free space*

μ_0, ϵ_0 constant $\Rightarrow$ realization of SI units of impedance.

Realization of the ampere

The (electrodynamic) ampere balance (Vigoreux, 1965)





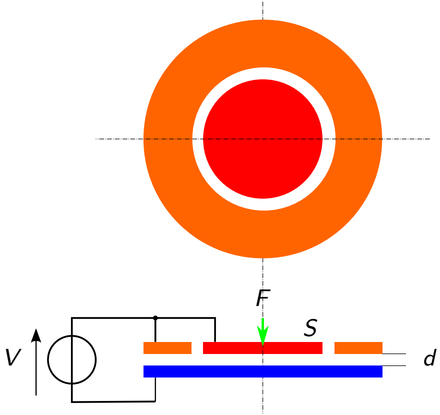
Ampère force law:

$$F = \frac{\mu_0}{4\pi} \int_{\Gamma_1} \int_{\Gamma_2} \frac{I_1 \, \mathrm{d}\boldsymbol{\ell}_1 \times I_2 \, \mathrm{d}\boldsymbol{\ell}_2 \times \boldsymbol{r}_{21}}{|\boldsymbol{r}_{21}|^2}$$

If $I_1 = I_2$, $F = \mu_0 k I^2$ where k is computed from geometrical measurements

Realization of the volt

The (electrostatic) voltage balance



Force between plates: $F = \epsilon_0 \dfrac{S}{2d^2} V^2 = \epsilon_0\, k\, V^2$

where k is computed from geometrical measurements

Realization of the volt

Cylindrical-electrode voltage balance, PTB (Siencknecht and Funck, 1986)

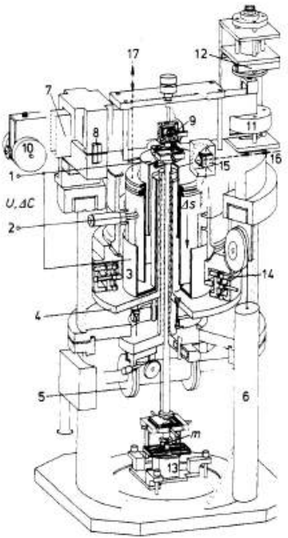
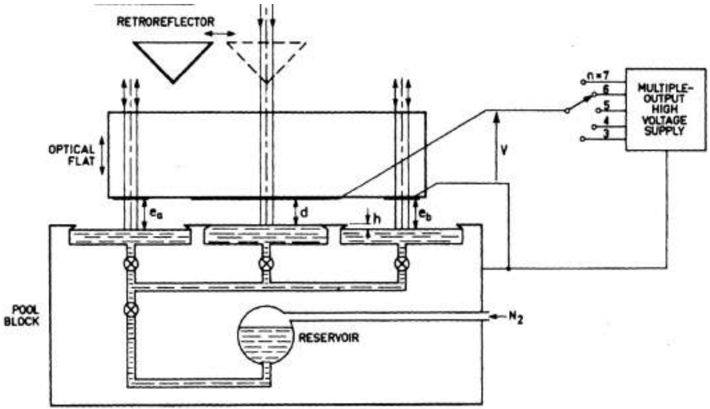


Fig. 1. Perspective view of the PTB voltage balance. 1 Inner electrode, 2 high-voltage electrode, 3 guard electrode, 4 carriage of displace unit, 5 driving device for displace unit, 6 counterweight of displace unit, 7 balance beam, 8 central joint of balance beam, 9 load joint of balance beam, 10 counterbalance weight, 11 position sensor, 12 retainer for balance beam, 13 load-changing device, 14 device for centering and vertical electrode adjustment, 15 interferometer for Δs-measurement, 16 light beam of interferometers for Δs-measurement, 17 light beam of autocollimator for vertical electrode adjustment

$V = 10\,186\,\mathrm{V} = 1000 \times E_{\mathrm{Weston}}$; $m = 2\,\mathrm{g}$!

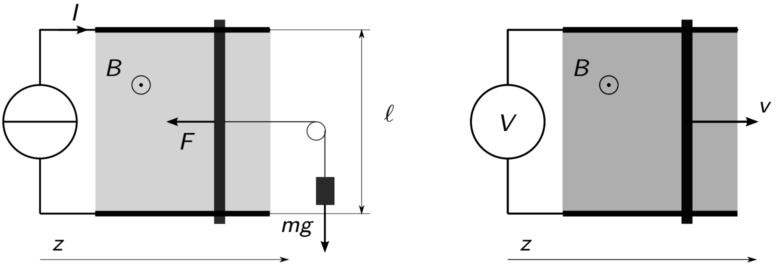
Realization of the volt

Mercury-electrode elevation, CSIRO Australia (Sloggett et al., 1985)



$V = \sqrt{\frac{2\rho g}{\epsilon_0}} d\sqrt{h}$. $V =$ kV, $d = 600\ \mu\text{m}$, $u_V = 0.33 \times 10^{-6}$

Solves the problem of geometrical measurements!



- Weighing mode: $F = B\ell I = \frac{\mathrm{d}\Phi}{\mathrm{d}z} I$
- Moving mode: $E = \frac{\mathrm{d}\Phi}{\mathrm{d}t} = \frac{\mathrm{d}\Phi}{\mathrm{d}z}\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\mathrm{d}\Phi}{\mathrm{d}z} v$
- $Fv = EI$; $P_{\mathrm{m}} = P_{\mathrm{e}}$

The Kibble balance

(Robinson and Schlamminger, 2016)

Solves the problem of geometrical measurements!

weighing mode

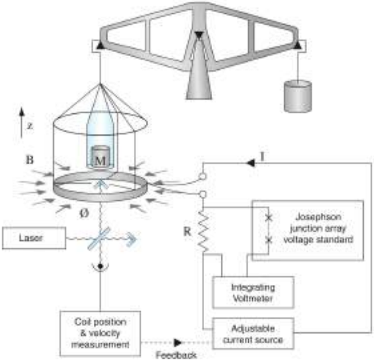


Figure 1. The Kibble balance in weighing mode.

moving mode

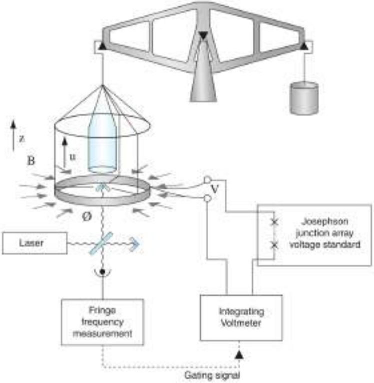
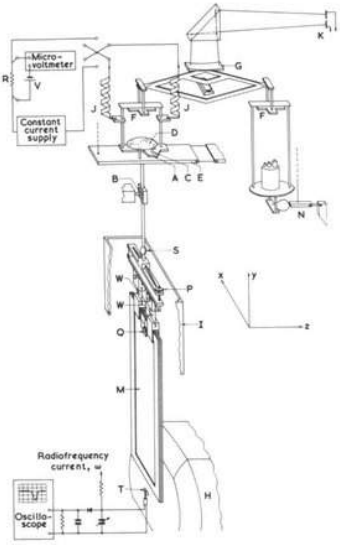


Figure 2. The Kibble balance in moving mode.

The Kibble balance evolution

NPL, Kibble (1976) for the gyromagnetic ratio of the proton

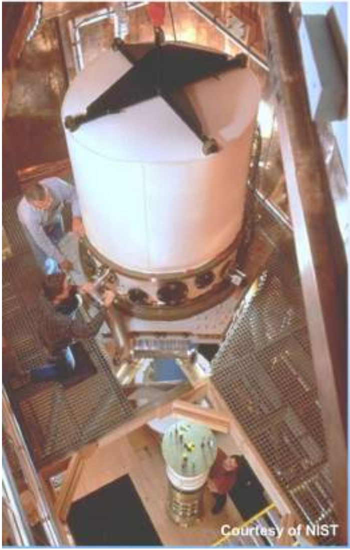


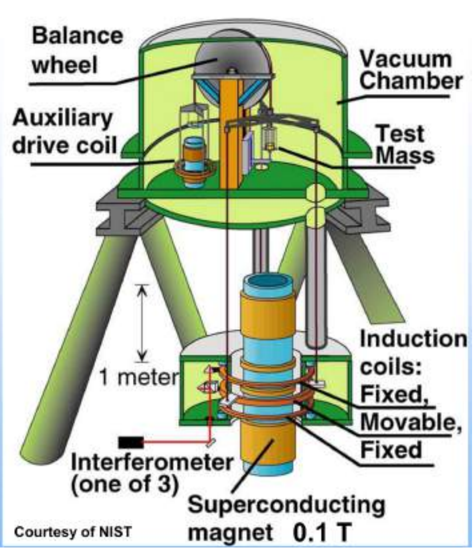
The Kibble balance: evolution

NRC, Bryan P. Kibble and I. Robinson, 2011

The Kibble balance: evolution

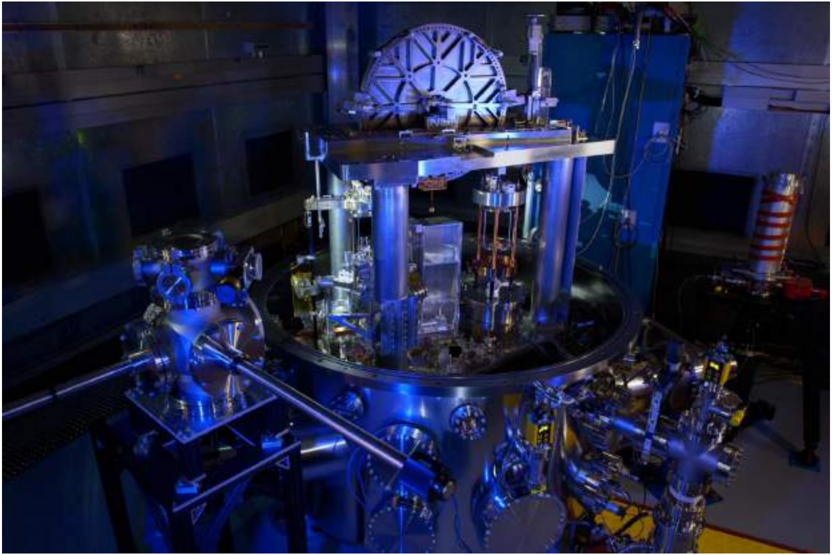
NIST-3





The Kibble balance: evolution

The next generation: NIST-4, 2016



The Kibble balance: evolution

The next generation: NPL, 2017

The Kibble balance

Determination of the Planck constant

To be discussed again after the quantum experiments

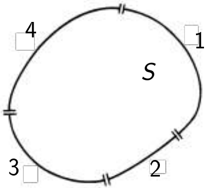
- $mgv = EI$
- $E = n\frac{f_\mathrm{E}}{K_\mathrm{J}}$
- $I = \frac{V_\mathrm{I}}{R} = \frac{f_\mathrm{I}}{K_\mathrm{J}}\frac{1}{rR_\mathrm{K}}$
- $K_\mathrm{J} = \frac{2e}{h}$
- $R_\mathrm{K} = \frac{h}{e^2}$

$$\Rightarrow mgv = hf_\mathrm{E}f_\mathrm{I}\frac{n}{r}$$

h can be measured mechanically

Realization of capacitance unit, the farad

the calculable capacitor



The general geometry of four conductors 1, 2, 3, 4 having cylindrical symmetry, and arranged in a closed shell with infinitesimal gaps, analyzed by the Thompson-Lampard theorem.

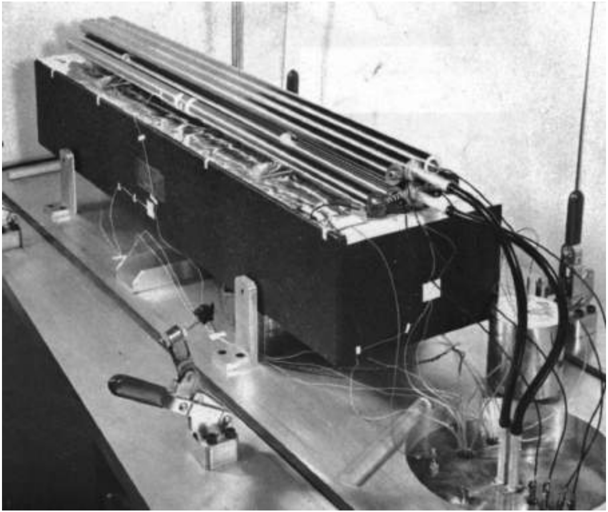
Thompson-Lampard theorem (Lampard, 1957)

$$\exp\left(-\pi\epsilon_0 C_{13}\right) + \exp\left(-\pi\epsilon_0 C_{24}\right) = 1.$$

If there is sufficient symmetry such that $C_{13} = C_{24} = C$,

$$C = \epsilon_0 \frac{\log 2}{\pi} = 1.953549043\ldots \times 10^{-12}\ \mathrm{F/m} \qquad \text{[exact]}.$$

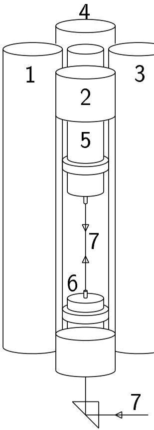
The calculable capacitor



1964: Fixed calculable capacitor, realized with stacked gauge bars, NRC (Dunn, 1964).

Realization of capacitance unit, the farad

the calculable capacitor



Cross capacitor with movable guard electrode. 1, 2, 3, and 4 are the four cylindrical electrodes to which the cross-capacitor theorem is applies. 5 and 6 are the two guard electrodes; electrode 6 can be moved axially between two positions; the motion is monitored by a laser interferometer 7.

$C = \epsilon_0 \frac{\log 2}{\pi} \ell$, where ℓ is a geometrical length to be measured.

The calculable capacitor

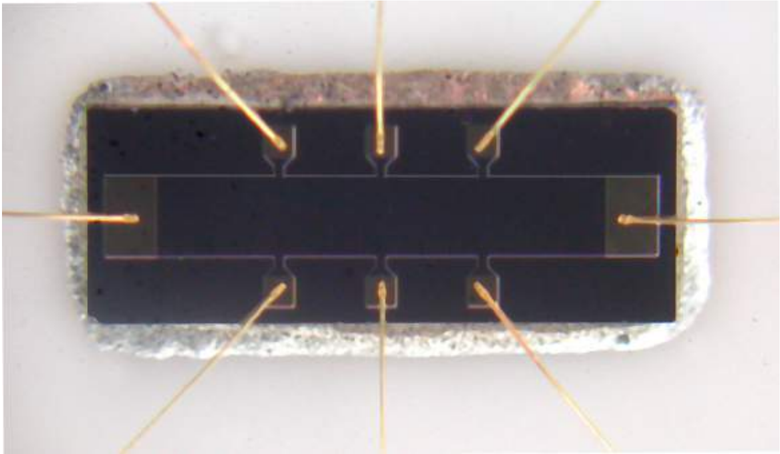
2015: NMIA-BIPM cross capacitor, with movable guard. (courtesy of J. Fiander)

Quantum electrical metrology experiments

Macroscopic quantum effect that display an electrical quantity related to fundamental constants

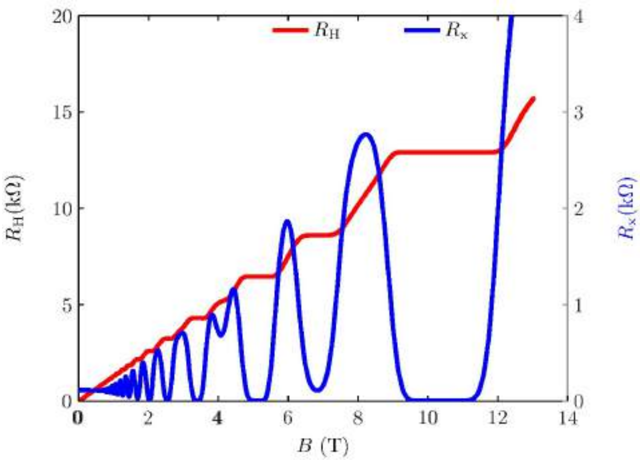
- quantized resistance: the quantum Hall effect
- quantized flux counting: the Josephson effect
- quantized charge counting: single-electron counting devices

The quantum Hall effect



AlGaAs/GaAs Hall bar heterostructure, 1 mm × 0.4 mm;

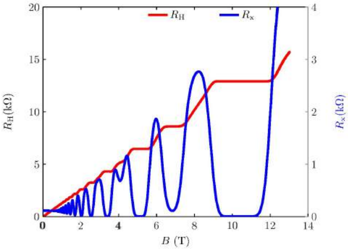
The quantum Hall effect



— $R_H = V_H/I$ Hall resistance;

— $R_x = V_x/I$ longitudinal resistance.

The quantum Hall effect

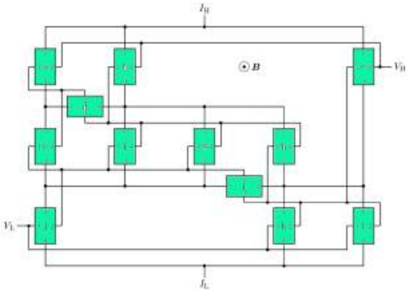


Each plateau i is centered on a resistance value $R_H = R_K/i$, with i integer

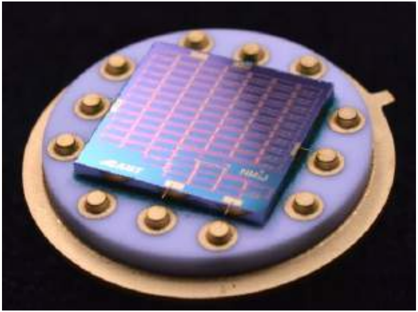
$$R_K = \frac{h}{e^2} = \frac{\mu_0 c}{2\alpha}.$$

R_K is linked to the fine structure constant α which can be measured by non-electrical means.

Quantum Hall array resistance standards



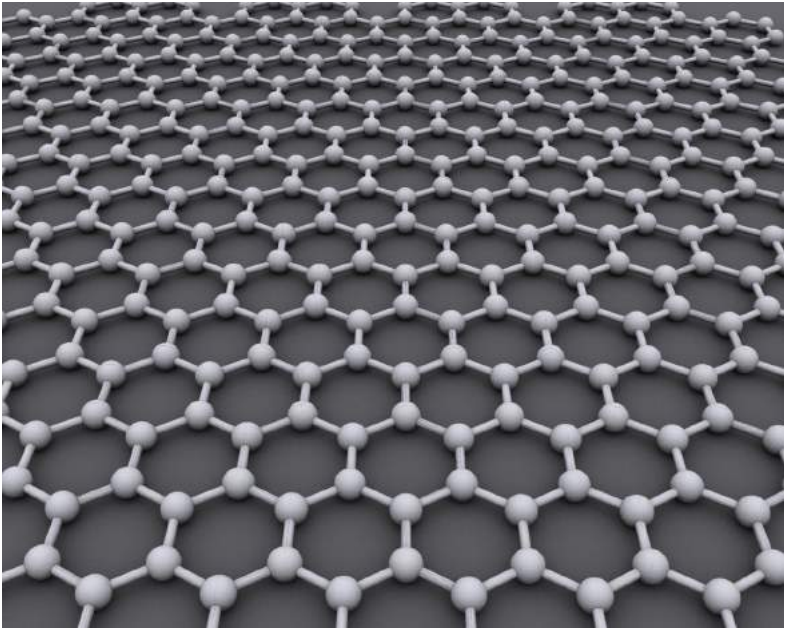
(a) 10 kΩ QHARS design (Ortolano et al., 2015)



(b) 1 MΩ QHARS (Oe et al., 2016)

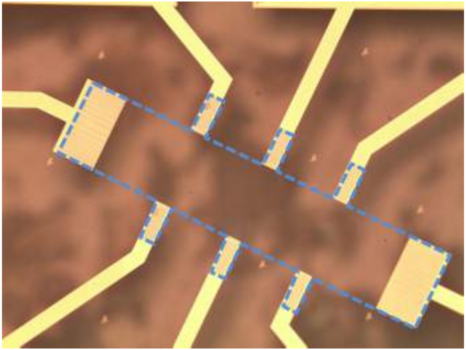
10 kΩ array: $R_{10\,\mathrm{k}\Omega} = \dfrac{203}{262} R_\mathrm{H} = (1 - 3.4 \times 10^{-8}) \times 10\,\mathrm{k}\Omega$

Graphene for QHE



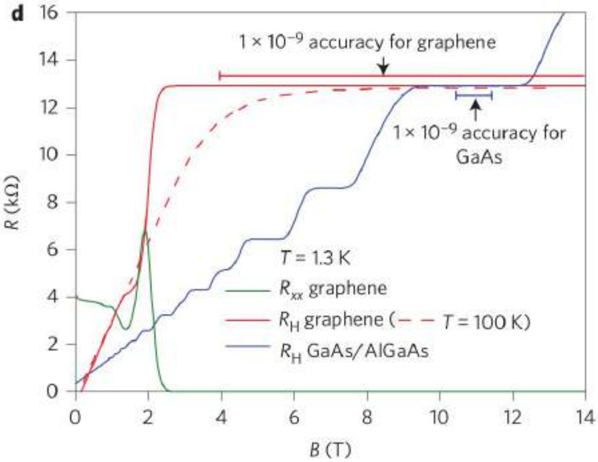
Graphene for QHE

PTB graphene Hall bar



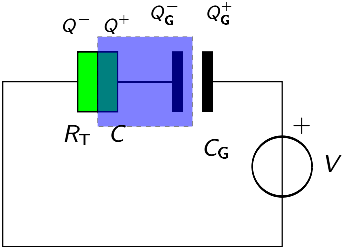
Courtesy: PTB

Graphene for QHE
(Ribeiro-Palau et al., 2015)

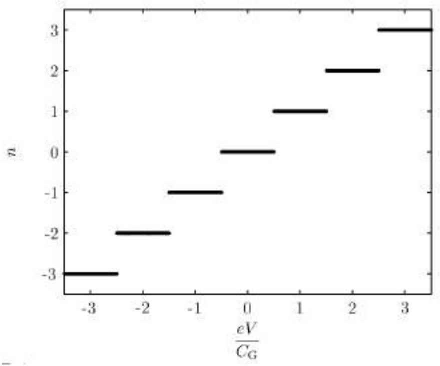


Quantized charge counting

Single charge confinement



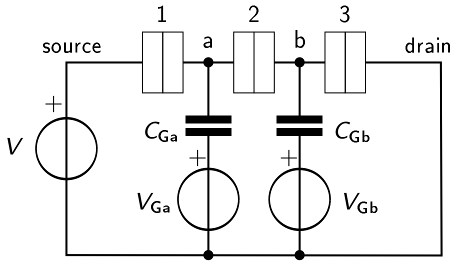
Single-electron box, coupled to an external circuit with a tunnel junction (with tunnel resistance R_T and capacitance C) and a capacitor C_G.



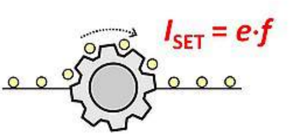
occupation number n versus applied bias voltage V.

Quantized charge counting

Nanodevices

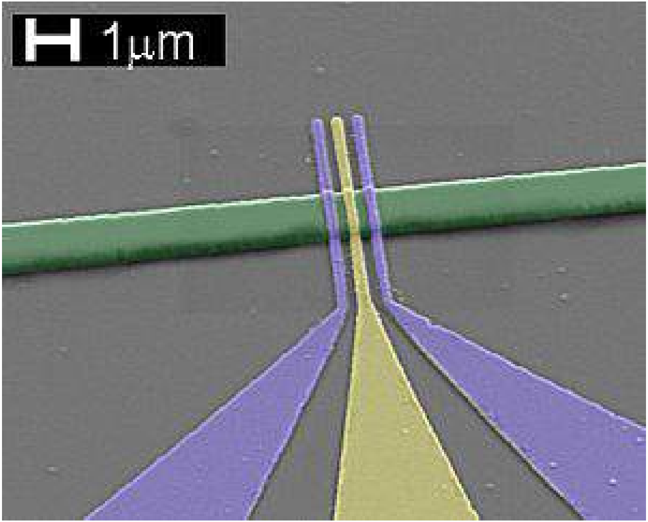


A three-junction single-electron pump.



Quantized charge counting

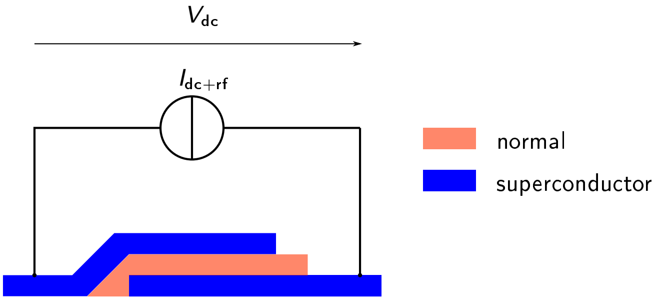
Nanodevices



Courtesy: PTB
Semiconductor single-electron pumps .

Counting flux quanta

Josephson junctions



Josephson junction:

- two superconductors coupled by a tunneling barrier
- have coupled wavefunctions

Counting flux quanta

frequency to voltage converter: the (inverse AC) Josephson effect

Under proper I_{rf} excitation amplitude of frequency f_{rf}

$$V_{\mathrm{dc}} = n\Phi_0 f_{\mathrm{rf}} = \frac{n}{K_{\mathrm{J}}} f_{\mathrm{rf}}$$

where

$\Phi_0 = h/2e = 2.067\,833\,831(13) \times 10^{-15}\,\mathrm{Wb}$ $[6.1 \times 10^{-9}]$ is the flux quantum;

$K_{\mathrm{J}} = 2e/h = 1/\Phi_0 = 483\,597.8525(30)\,\mathrm{GHz/V}$ is the Josephson constant;

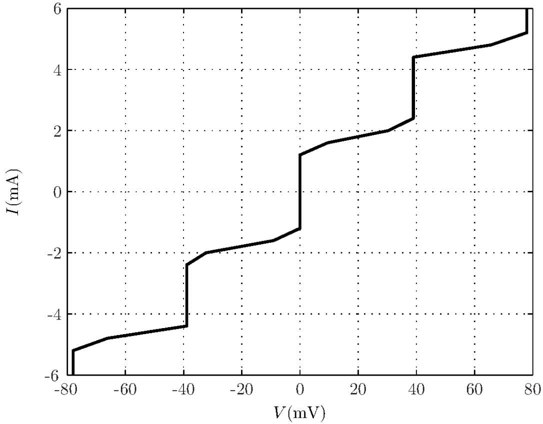
n is a small integer.

Feasible drive frequencies:

$f_{\mathrm{rf}} = 70\,\mathrm{GHz} \quad \Rightarrow \quad V_{\mathrm{dc}} = 150\,\mu\mathrm{V}$.

Counting flux quanta

frequency to voltage converter: the (inverse AC) Josephson effect

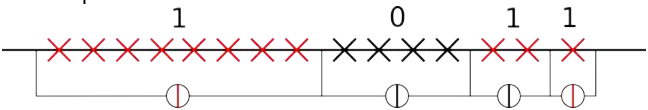


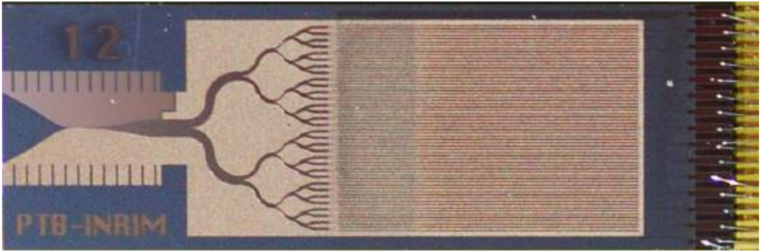
The $I - V$ characteristic of a Josephson array (256 junctions) under microwave irradiation. Steps $n = 0, \pm 1, \pm 2$ are visible. $f \approx 73\,\mathrm{GHz}$

Counting flux quanta

Josephson binary DAC

Binary-weighted Josephson DAC





Josephson junction binary array chip. 13 bit+sign DAC with 8192 superconducting-normal metal-insulator-superconductor (SNIS) junctions. The junctions are geometrically arranged over 32 parallel strips of 256 junctions each. $f = 70\,\text{GHz}$. $V_{\text{fullscale}} \approx \pm 1.2\,\text{V}$

The quantum experiments in the framework of the present SI

Knowledge in 1989 (CODATA):

$K_J = 483\,597.9(2)\,\text{GHz/V} \qquad [4 \times 10^{-7}]$

$R_K = 25\,812.807(5)\,\Omega \qquad [2 \times 10^{-7}]$

but, *reproducibility* of Josephson and quantum Hall experiments in different experiments and different laboratories was much higher: 10^{-9}–10^{-10}

Solution: invent non-SI units! 18th CGPM resolution 6: Valid since January 1, 1990:

$K_{J\text{-}90} = 483\,597.9\,\text{GHz/V} \qquad [\text{exact}]$

$R_{K\text{-}90} = 25\,812.807\,\Omega \qquad [\text{exact}]$

To $K_{J\text{-}90}$ and $R_{K\text{-}90}$ the conventional units Ω_{90}, H_{90}, F_{90}, A_{90}, W_{90} are associated.
These are the electrical units in use nowadays.

Becuase of improvements in the measurement of fundamental constants, today (CODATA 2014)

$$K_\mathrm{J} = 483\,597.8525(30)\,\mathrm{GHz/V} \qquad [6.1 \times 10^{-9}]$$

$$R_\mathrm{K} = 25\,812.807\,455\,5(59)\,\Omega \qquad [2.3 \times 10^{-10}]$$

Therefore

$$V_{90} = 1 + 9.8(6) \times 10^{-8}\,\mathrm{V}$$

$$\Omega_{90} = 1 - 1.764(2) \times 10^{-8}\,\Omega$$

$\Rightarrow$ Unacceptable deviation of the conventional units respect to the SI units

The Kibble balance

Determination of the Planck constant

Now the derivation can be clarified

- $mgv = EI$
- $E = n \dfrac{f_\mathrm{E}}{K_\mathrm{J}}$
- $I = \dfrac{V_\mathrm{I}}{R} = \dfrac{f_\mathrm{I}}{K_\mathrm{J}} \dfrac{1}{rR_\mathrm{K}}$
- $K_\mathrm{J} = \dfrac{2e}{h}$
- $R_\mathrm{K} = \dfrac{h}{e^2}$

$$\Rightarrow mgv = hf_\mathrm{E} f_\mathrm{I} \frac{n}{r}$$

h can be measured mechanically

The forthcoming SI
The seven base units

The SI is the system of units in which:

- s The unperturbed ground state hyperfine transition frequency of the caesium 133 atom $\Delta\nu_{\mathrm{Cs}}$ is 9 192 631 770 Hz;
- m the speed of light in vacuum c is 299 792 458 m/s;
- kg the Planck constant h is $6.626\,070\,15 \times 10^{-34}$ J s;
- A the elementary charge e is $1.602\,176\,634 \times 10^{-19}$ C;
- K the Boltzmann constant k is $1.380\,649 \times 10^{-23}$ J/K;
- mol the Avogadro constant N_{A} is $6.022\,140\,76 \times 10^{23}$ mol^{-1};
- cd the luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz, K_{cd}, is 683 lm/W,

where the hertz, joule, coulomb, lumen, and watt, with unit symbols Hz, J, C, lm, W, respectively, are related to the units second, metre, kilogram, ampere, kelvin, mole, and candela, with unit symbols s, m, kg, A, K, mol, cd, respectively, according to $\mathrm{Hz} = \mathrm{s}^{-1}$, $\mathrm{J} = \mathrm{m}^2\mathrm{kg}\mathrm{s}^{-2}$, $\mathrm{C} = \mathrm{A\,s}$, $\mathrm{lm} = \mathrm{cd\,sr}$, $\mathrm{W} = \mathrm{m}^2\mathrm{kg}\mathrm{s}^{-3}$.

The forthcoming SI

Redefinition of the SI base of interest for electromagnetism:

kg the kilogram;

A the ampere;

by fixing the values of the fundamental constants:

h Planck constant;

e elementary charge;

The forthcoming SI: the base unit ampere

The ampere will be redefined as:

> *The ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge e to be* $1.602\,176\,634 \times 10^{-19}$ *when expressed in the unit C, which is equal to A s, where the second is defined in terms of* $\Delta\nu_{\mathrm{Cs}}$.

The kilogram will be redefined as:

> *The kilogram, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant h to be* $6.626\,070\,15 \times 10^{-34}$ *when expressed in the unit J s, which is equal to* $\mathrm{kg\,m^2\,s^{-1}}$, *where the metre and the second are defined in terms of c and* $\Delta\nu_{\mathrm{Cs}}$.

The forthcoming SI: realization of the units

Consequences of the redefinition

e will be **exact**;

$\Rightarrow$ any electron-counting experiment will be a realization of the ampere;

$R_K = \dfrac{h}{e^2}$ will be **exact**;

$\Rightarrow$ the quantum Hall effect will be a realization of the ohm;

$K_J = \dfrac{2e}{h}$ will be **exact**;

$\Rightarrow$ the Josephson effect will be a realization of the volt;

$\Rightarrow$ Combining Josephson and quantum Hall effects with Ohm's law will be a realization of the ampere.

The forthcoming SI: electromagnetic fundamental constants

μ_0 the magnetic constant will be no more $4\pi \times 10^{-7}$ H/m: not exact and subject of measurement;

$\epsilon_0 = \frac{1}{\mu_0 c^2}$ the electric constant will be no more exact;

$\Rightarrow$ ϵ_0 and μ_0 will have the same relative uncertainty and will be totally correlated (correlation coefficient $= -1$)

$Z_0 = \mu_0 c$ the impedance of free space, and

$Y_0 = (\mu_0 c)^{-1}$ the admittance of free space will be no more exact;

The forthcoming SI: realization of the units

A new role for the mechanical experiments

h will be **exact**;

$\Rightarrow$ The Kibble balance, if traceable to K_J and R_K, will be a realization of the kilogram.

Same for the voltage and the current balances

The forthcoming SI: mise en pratique

Draft for Appendix 2 of the SI Brochure for the "Revised SI" 8/12/2017
Version 1.0

Mise en pratique for the definition of the ampere and other electric units in the SI

Consultative Committee for Electricity and Magnetism

Linking SI-90 with SI

8/12/2017
Version 1.0

CCEM Guidelines for Implementation of the 'Revised SI'

Consultative Committee for Electricity and Magnetism

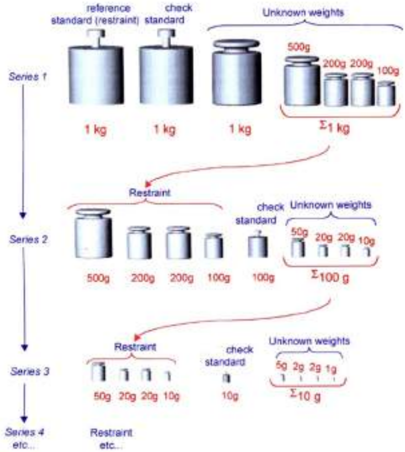
- $V_{90} \Rightarrow V$: $d = +1.067 \times 10^{-7}$
- $\Omega_{90} \Rightarrow \Omega$: $d = +1.779 \times 10^{-8}$

$d < 2.5\,U$: no action until next recalibration

$d > 2.5\,U$: numerical correction to be applied

The forthcoming SI: benefits

Any physical experiment that satisfies the definition is a realization of the unit;



PAPER

Milligram mass metrology using an electrostatic force balance

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Metrologia, Volume 53, Number 5

Focus on Realization, Maintenance and Dissemination of the New Kilogram

Units can be realized at any level (multiple, submultiple)

The CODATA 2017 adjustment of the fundamental constants

Minimum change of the units size

The CODATA 2017 Values of h, e, k, and N_A for the Revision of the SI*

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(Dated: July 24, 2017)

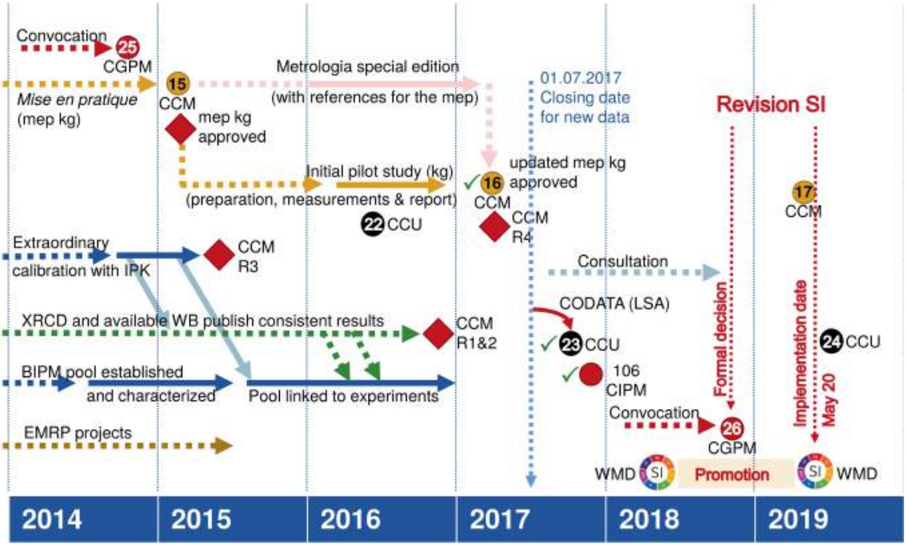
TABLE II The CODATA 2017 adjusted values of h, e, k, and N_A

Quantity	Value	Rel. stand. uncert u_r
h	$6.626\,070\,147(67) \times 10^{-34}$ J s	1.0×10^{-8}
e	$1.602\,176\,6338(81) \times 10^{-19}$ C	5.1×10^{-9}
k	$1.380\,649\,01(51) \times 10^{-23}$ J K^{-1}	3.7×10^{-7}
N_A	$6.022\,140\,761(61) \times 10^{23}$ mol^{-1}	1.0×10^{-8}

TABLE III The CODATA 2017 values of h, e, k, and N_A for the revision of the SI

Quantity	Value
h	$6.626\,070\,15 \times 10^{-34}$ J s
e	$1.602\,176\,634 \times 10^{-19}$ C
k	$1.380\,649 \times 10^{-23}$ J K^{-1}
N_A	$6.022\,140\,76 \times 10^{23}$ mol^{-1}

The roadmap towards the new SI



Formal decision: the CGPM

26th General Conference of Weights and Measures

26^{e} CGPM
Versailles
13-16 novembre 2018

May 20, 2019

World Metrology Day

Stay prepared!

Further reading

- "Draft of the 9th SI brochure," 5 Feb 2018
- CCEM Working Group on the SI, "Mise en pratique for the ampere and other electric units in the international system of units," 2017, CCEM-17-08
- P. J. Mohr, D. B. Newell, and B. N. Taylor, "CODATA recommended values of the fundamental physical constants: 2014," *J. Phys. Chem. Ref. Data*, vol. 45, 2016
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